Aeroelastic Divergence of Trimmed Aircraft

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INTEREST in the correct model conditions for the calculation of the aeroelastic divergence speeds of nominally unconstrained aircraft has risen with the interest in aircraft with forward-swept wings.[1,2] Often zero-frequency solutions of oscillatory equations are taken to give free-free divergence speeds, but there is a danger that in so doing assumptions which are not entirely appropriate are carried over from an oscillatory case to what is essentially a static case. In this Note, the equations of a simple aircraft in an idealized trimmed shallow pullout, which can be derived from statics, are studied in the belief that the speed at which they become indeterminate provides a suitable definition of the "unconstrained" divergence speed of the aircraft. This speed is found to be high if compared to the fixed-root divergence speed (FDS), but the equations also show both that the angle of aircraft pitch will be negative at speeds above a speed near the FDS, and so only the incidence due to torsion contributes positive lift, and that there might be control difficulties at speeds near the lower divergence speed.

In the interest of clarity, the wing considered is unswept and rectangular, but forward swept and tapered wings can be treated in a similar manner. Further, factors that are important in the design of actual aircraft (such as the effect of the downwash from one surface on the airflow over another), but not thought essential to the present purpose are omitted. The aircraft is taken to consist of a rectangular wing, flexible in torsion, fixed to a rigid, but otherwise unconstrained, thin fuselage to which is also fixed a rectangular trimming surface, rigid in itself but hinged in the spanwise direction at its aerodynamic center. The aircraft is taken to be stationary in an airflow as if it were in a wind tunnel, but to be accelerating in heave. Some account is taken of the pitching velocity implied by a pullout.

The generalized coordinates q_h, q_p, q_t, and q_s associated with the degrees of freedom are shown in Fig. 1 in which the trimming surface is a foreplane. q_h is the generalized coordinate of heave, measured at the aerodynamic center of the trimming surface, with the chord of the wing c as the reference length; q_p the coordinate of pitch of the aircraft as a rigid body; q_t the coordinate of the wing torsion mode; and q_s the coordinate of the trimming surface angle, relative to the fuselage, necessary to trim the aircraft.

The equations for the trimmed shallow pullout can be obtained by such as the method of virtual work and written as

$$\frac{\frac{1}{2}\rho v^2 Sa_1}{Mg}\begin{bmatrix} c_{hp} & c_{ht} & c_{hs} \\ c_{pp} & c_{pt} & 0 \\ c_{tp} & c_{tt}(1-v^{-2}) & 0 \end{bmatrix}\begin{bmatrix} q_p \\ q_t \\ q_s \end{bmatrix} = \begin{bmatrix} n+1+nb \\ (n+1)\bar{\xi} \\ (n+1)w\xi_i\int\theta \end{bmatrix} \tag{1}$$

where $\frac{1}{2}\rho V^2 Sa_1$ is the lift on the rigid wing when its incidence is unity, Mg the weight of the whole aircraft, and $\bar{\xi}$ the scaled distance its c.g. is aft of the aerodynamic center of the trimming surface. The c_{ij} coefficients represent the aerodynamic forces and v the airspeed as a fraction of the fixed-root divergence speed. As a consequence of this choice of speed scale, the structural stiffness coefficient for the wing torsion mode is equal and opposite to c_{tt}. The right-hand side of Eq. (1) is a column of the generalized forces due to both gravity and a normal acceleration of ng.

The first row of the equation equates the total lift on the aircraft, which is the sum of lifts due to aircraft pitch, wing torsion, and trimming surface incidence, with the sum of its weight, normal inertia force, and b (the aerodynamic damping force on the trimming surface due to the pitching velocity). The second row equates the aerodynamic pitching moment with the gravity and normal acceleration pitching moments. These pitching moments are about the aerodynamic center of the trimming surface and so the aerodynamic forces on the trimming surface do no work in the coordinate. The third row equates the sum of the aerodynamic torsional and structural stiffness moments about the wing flexural axis with the twisting moment due to self-weight and normal acceleration (w is the mass of the wing as a fraction of the mass of the whole aircraft, ξ_i the scaled uniform distance the wing inertia axis is aft of its flexural axis, and $\int\theta$ represents the integral over the wing span of the torsion mode shape). Note that, although there are four generalized coordinates, the coefficient matrix is only third order since there is no equation in trimming surface angle because the aircraft is constrained to be in trim and there are no forces due to displacement in the heave coordinate.

These trim equations have a solution at all speeds apart from that for which the value of v makes the coefficient matrix singular and hence the amplitudes of the generalized coordinates q indeterminate. This speed is

$$v = [1 - c_{pt}c_{tp}/(c_{pp}c_{tt})]^{-\frac{1}{2}} \tag{2}$$

Equation (2) owes its simplicity to the prescient choice of pitching axis. By analogy with the fixed-root case this speed can be called the divergence speed of the trimmed aircraft (TDS). If the incidences associated with q_p and q_t are taken as positive in the same sense, which introduces no restriction, and the pitching axis is at the trimming surface, c_{pp} and c_{pt} share a common sign as do c_{tp} and c_{tt}, whose signs are independent of the position of the pitching axis; v is greater than unity and the TDS is higher than the FDS. If two-dimensional derivatives are assumed in the present case, it can be shown that the torsion mode at the TDS is a half-wave of $1-\cos\alpha$, rather than the quarter-wave of $\sin\alpha$ that is appropriate at the FDS, and the TDS is twice the FDS.

A free-free divergence speed can be derived from the solution of flutter equations at zero frequency. An equation similar to Eq. (2) is obtained, but the pitching axis used in flutter equations is likely to be that which runs through the c.g. of the aircraft, in which case c_{pp} is a scaled pitching moment of the rigid aircraft about its c.g. and has the same sign as the "tail-on" c.g. margin of the rigid aircraft. Because the present wing is rectangular, the chordwise position of the center of action of the wing lift due to torsion is coincident with that of the wing lift due to pitch and c_{pt}, which depends only on the aerodynamic forces on the wing, has the same sign as the

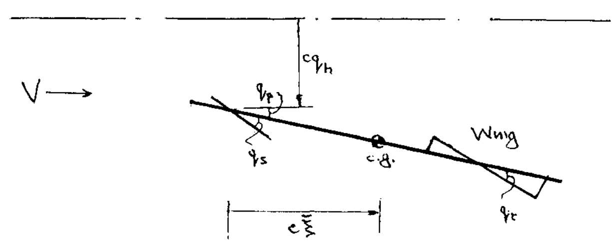

Fig. 1 Generalized coordinates for aircraft displacements.

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"tail-off" c.g. margin, while c_{tp} and c_{tt} share the same sign, as before. Thus, when the pitching axis runs through the aircraft c.g., v is greater or less than unity according to whether the two c.g. margins have the same sign or not; but when the "tail-on" c.g. margin is infinite and the "tail-off" c.g. margin is finite, v is unity.

To help in the study of the conditions at and near the FDS, we differentiate Eq. (1) with respect to n, put $\nu=1$, and, to simplify matters, assume that the wing is mass balanced ($\xi_i=0$) and get

$$\begin{bmatrix} c_{hp} & c_{ht} & c_{hs} \\ c_{pp} & c_{pt} & 0 \\ c_{tp} & 0 & 0 \end{bmatrix} \begin{bmatrix} q_e' \\ q_t' \\ q_s' \end{bmatrix} = \begin{bmatrix} 1+b \\ \bar{\xi} \\ 0 \end{bmatrix} \tag{3}$$

where $q' = \mathrm{d}q/\mathrm{d}n$ and $\alpha_0 = Mg/(\frac{1}{2}\rho V^2 S a_1)$.

This matrix equation can be solved by inspection to get

$$q_p' = 0 \tag{4a}$$

$$q_t'\alpha_0 = \bar{\xi}/c_{pt} \tag{4b}$$

$$q_s'/\alpha_0 = 1 - \bar{\xi}c_{ht}/c_{pt} + b \tag{4c}$$

Under the same conditions, Eq. (1) gives the pitch angle itself as zero[3] and, from this and Eq. (4a), we see that the pitch angle is zero whatever the normal acceleration. All the wing incidence is due to torsion and in the present case the incidence at the wing tip is 50% greater than it would be were the wing rigid. Hence, the spanwise position of the center of action of the wing lift is further outboard than it would be on a rigid wing and the flexural and torsional moments the wing structure has to resist are increased. The maldistribution of lift over the span becomes worse as the speed is increased above the FDS, because the pitch angle becomes increasingly negative and the amount of torsion must be increased to counteract it.

Turning our attention to Eq. (4c), the trimming surface angle, per g, c_{pt}/c_{ht} is the moment arm in pitch of the lift due to torsion that, in this case, is the distance between the aerodynamic centers of the trimming surface and wing. Hence $\bar{\xi}c_{ht}/c_{pt}$ will be greater or less than unity according to whether the "tail-off" c.g. margin is negative or positive, but in either case $1-\bar{\xi}c_{ht}/c_{pt}$ is likely to be small. b, the trimming surface pitch damping, is $\frac{1}{2}\rho sc\bar{\xi}a_1/M$, where s is the area of the trimming surface and the other letters have their previous meanings,[4] and is also likely to be small. Thus, the trimming surface angle per g will be small at the FDS and will be zero for a particular speed near it and at this speed the aircraft will be in trim whatever the normal acceleration.

The foregoing describes an essentially static conception of divergence and the aeroelastic phenomena allied to it. The divergence speed given by the trim equations is high, but the same equations show that there might be problems due to adverse spanwise loadings at lower speeds and that, at speeds near its fixed-root divergence speed, the normal acceleration of the aircraft will be very sensitive to trimming surface angle. The example taken is simple, but there seems no reason why similar results would not be obtained from more comprehensive equations.

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Dynamic Loads on Twin Jet Exhaust Nozzles Due to Shock Noise

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Introduction

STRUCTURAL failure of the B-1 aircraft exhaust nozzle external flaps has been observed during flight tests.[1] Similar damage to some F-15 aircraft has also been noted. Since both aircraft contain twin engines with similar internozzle spacing a phenomenon resulting from the interaction between the two jets is a possible cause of this damage. This Note examines the acoustic near field generated by model single and twin jet configurations to determine if closely spaced dual nozzles generate significantly higher acoustic loading than that encountered with a single nozzle.

The major component of the noise from a jet measured in the downstream direction is due to turbulent mixing. However, for underexpanded sonic jets and imperfectly expanded supersonic jets, the sound radiated in the upstream direction is dominated by shock noise, consisting of both random and discrete components designated as broadband shock noise and jet screech, respectively.[2] Screech is a feedback process between the shock cells and the nozzle exit involving disturbances within the jet and upstream traveling sound waves. It is characterized by stages, with stage changes being marked by a frequency shift in the generated tones[3] and a spatial shift in the downstream shock structure.[4] As many as five stages have been identified, these having been labeled[5] as stages A_1, A_2, B, C, and D. The maximum amplitude of the fundamental tone of each stage is measured in the upstream direction.[4] For a given nozzle pressure ratio, increasing jet temperature raises the screech frequency due to higher velocities in the jet.[6] Flight influences which stage dominates[7] and lowers screech frequencies by increasing the feedback cycle time.[8] Screech tones have been measured in a Trident aircraft in flight[9] and have been blamed for structural damage to the empennage of a VC10 aircraft.[10]

Experiment

The experiment was conducted in the NASA Langley Quiet Flow Facility.The dual nozzles were constructed from ⅝ in. internal diameter pipe. Their external surfaces were machined to provide a thin lip at the nozzle exit and a smooth transition to the housing in which they were mounted (Fig. 1). The nozzle spacing was chosen to equal that of a scaled B-1 aircraft (approximately 1.9 nozzle diam). A ¼ in. microphone was strapped to the interfairing of the nozzles to measure the acoustic field near the nozzle exit

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